

# Broadband free-space characterization of metamaterials

M. H. Belyamoun, O. Dubrunfaut, C. Pareige, Y. Zhu, S. Zouhdi, and F. Ossart

Laboratoire de Génie Électrique de Paris

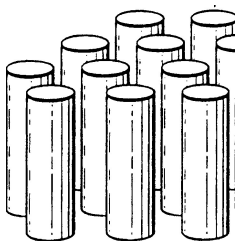


PIERS 2011

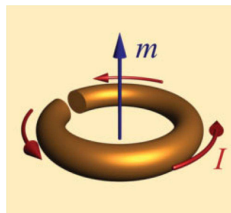
# Metamaterials

Artificial periodic structures with exotic electromagnetic properties at the macroscopic scale.

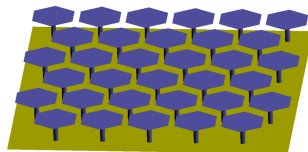
- a) Epsilon-negative rods.
- b) **S**plit **R**ing **R**esonator (negative permeability)
- c) **H**igh **I**mpedance **S**urface : Sievenpiper mushrooms.



a



b

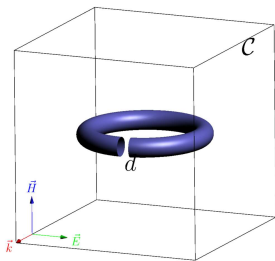
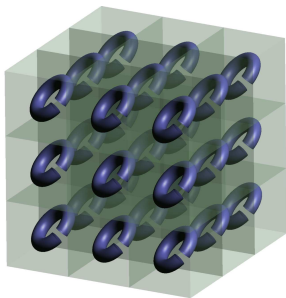


c

# Summary

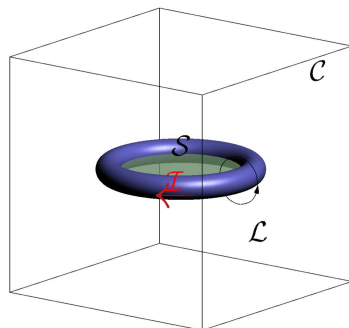
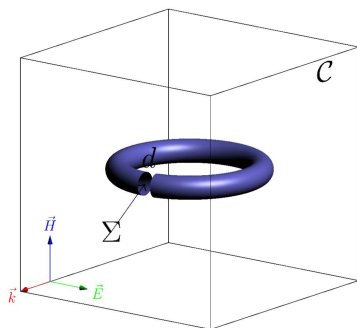
- 1 Introduction
- 2 Numerical model
  - SRR homogenization
  - HIS homogenization
- 3 The free space characterization system
  - A focused system
  - Thru Reflect Line calibration
  - Time domain analysis
- 4 Free space measurements of metamaterials
  - Shifted metallic wire rods
  - High Impedance Surface
  - SRR characterization
- 5 Conclusions and outlook

# Reference cell



- The SRR acts as an LC resonator.
  - We want to compute the effective permeability :  $(\vec{B} = \mu_{eff} \vec{H})$ .
- Incident plane wave.
  - Long wavelength  $\lambda \gg a$
  - Perfect conductor :  $h = 0$  inside the ring.

# From the split-ring to a closed ring



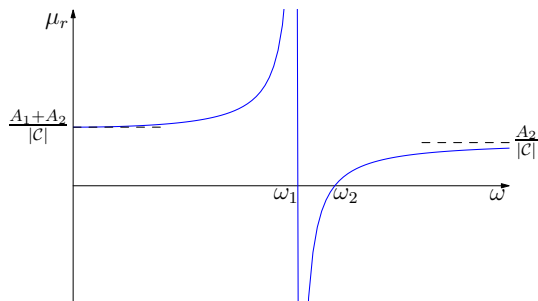
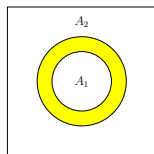
## Cell energy

$$|\mathcal{C}| H \bar{\mu}_{\text{eff}} \bar{H} = \int_{\mathcal{A}} \mu |\nabla \varphi|^2 + \int_{\partial \mathcal{R}} \frac{1-i}{\sigma \omega \delta} |\nabla_S \varphi|^2 - \frac{1}{C \omega^2} \mathcal{I}^2$$

where  $\varphi$  stands for the magnetic potential.

# Polaritonic law for 2D-structures

- $h = h_1$  over  $A_1$  and  $h = h_2$  over  $A_2$
- Faraday :  $i\omega\Phi + V = 0$  and Ampère :  $\mathcal{I} = h_1 - h_2$ .
- Magnetic field flux :  $|C|B = \mu (|A_1|h_1 + |A_2|h_2)$  and  $\Phi = \mu h_1 |A_1|$
- Capacitor in the air gap :  $\mathcal{I} = i\omega CV$ .

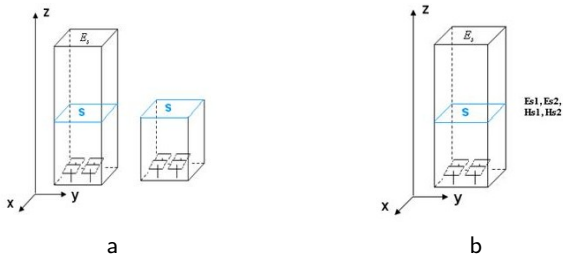


$$\mu_{\text{eff}} = \mu \frac{|A_1| + |A_2|}{|C|} \frac{\left(\frac{\omega}{\omega_2}\right)^2 - 1}{\left(\frac{\omega}{\omega_1}\right)^2 - 1}$$

$$\omega_1 = (\mu C |A_1|)^{-\frac{1}{2}}$$

$$\omega_2 = \sqrt{1 + |A_1|/|A_2|} \omega_1$$

# Effective impedance computation



- Symmetric HIS :
  - ▶ Impedance extension from 1D  $\rightarrow$  2D

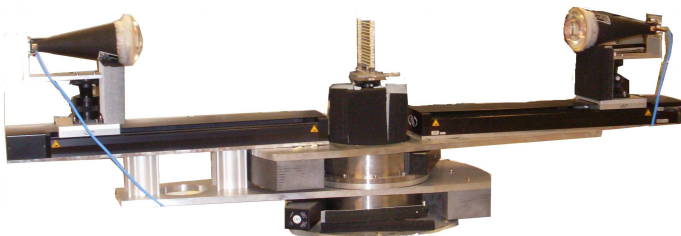
- Asymmetric HIS :

a) Flux method  $\int_C i\omega(\epsilon E_i E_j - \mu H_i H_j) = |S| E'_{js} \frac{1}{Z_{ij}} E_{is}$

- b) Surface impedance method

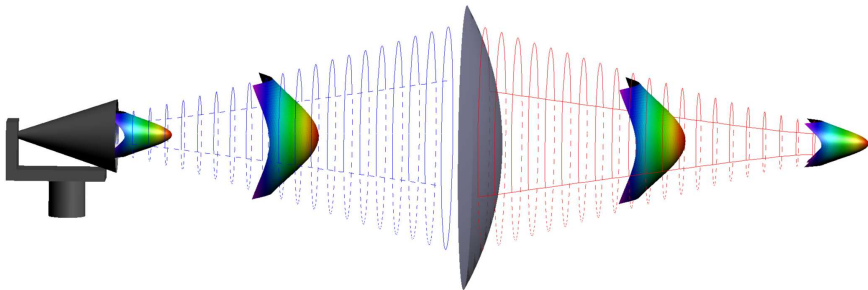
$$\begin{pmatrix} -H_{1sy} & -H_{2sy} \\ H_{1sx} & H_{2sx} \end{pmatrix} = - \begin{pmatrix} Y_{xx} & Y_{xy} \\ Y_{yx} & Y_{yy} \end{pmatrix} \begin{pmatrix} E_{1sx} & E_{2sx} \\ E_{1sy} & E_{2sy} \end{pmatrix}$$

# The free-space characterization system



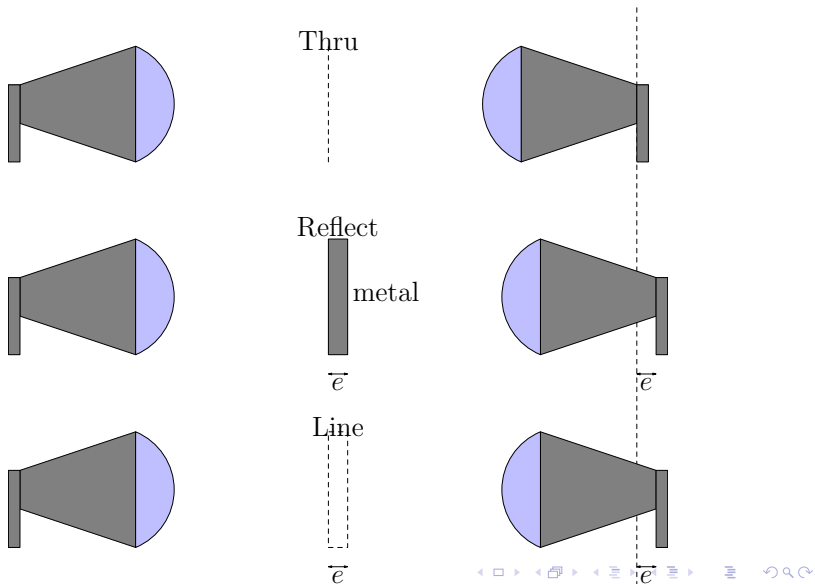
- 2 quad-ridged horn antennas (2-18 GHz), mounted on micrometric stages.
- The beam is focused with Rexolite ( $\epsilon_r = 2.54$ ) lenses.
- Agilent PNA 8364C : S-parameters measurement.

# Gaussian beam

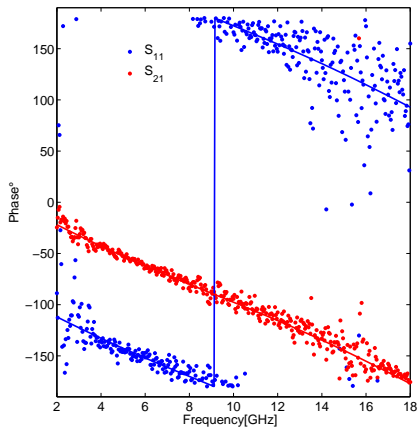
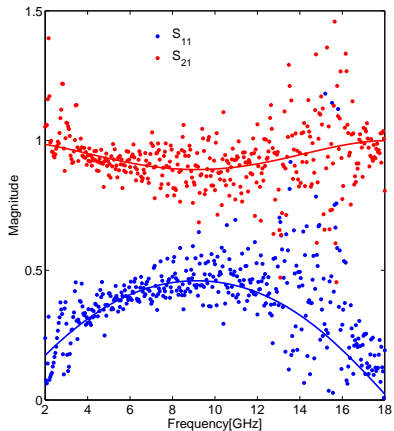


- Gaussian beam.
  - We must have an incident planar wave on the sample to compute the effective parameters.
  - Negligible diffraction if the sample dimension is 3 times the beam's radius.
- A focused bench is much smaller.

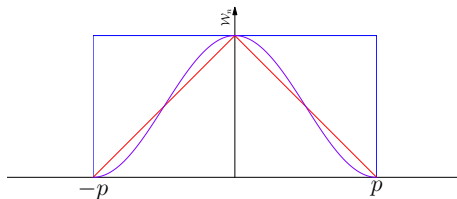
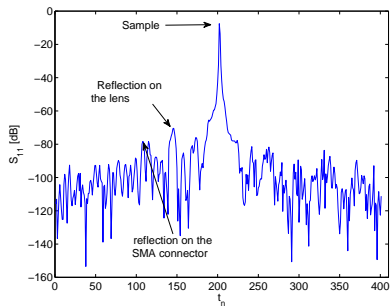
# Thru Reflect Line calibration



# S parameters of a plexiglas sample ( $e=5$ mm)



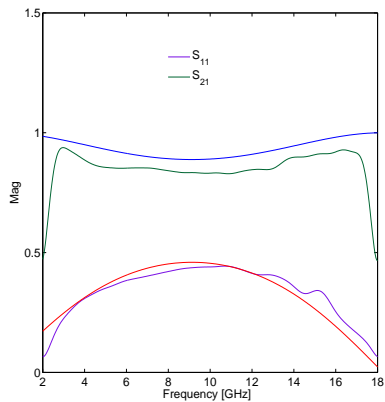
# Time domain analysis



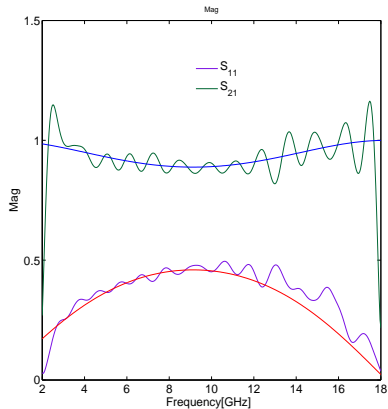
- Hanning
- Triangular
- Rectangular

- The calibrated S parameters are transferred to the time domain.
- A window is applied to eliminate the parasite reflections.
- Gibbs phenomenon limits the selectivity of the filter.

# Influence of the filter type (Plexiglas, $e = 5$ mm)

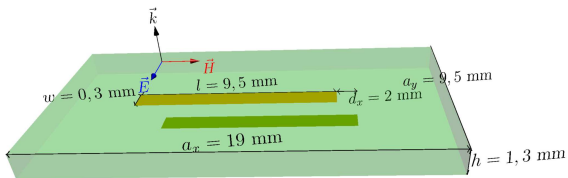


Hanning

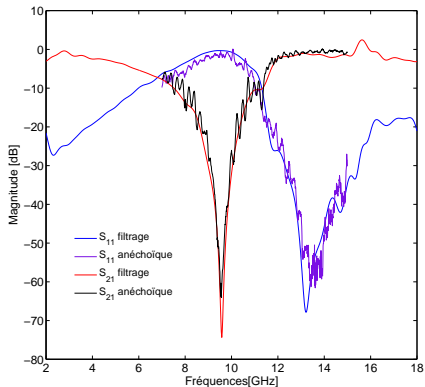


Window

# Shifted metallic wire rods



- Characterization in an anechoic chamber.
- Time domain with a Hamming window.

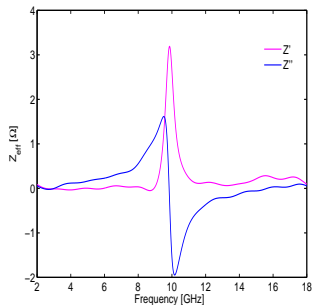
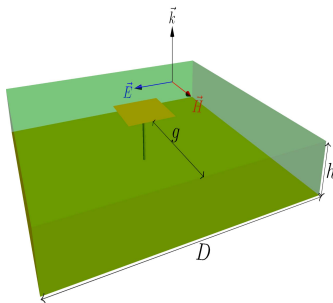


# HIS characterization

Epoxy  $\epsilon_r = 4,4$   
 $h = 2.4$  mm

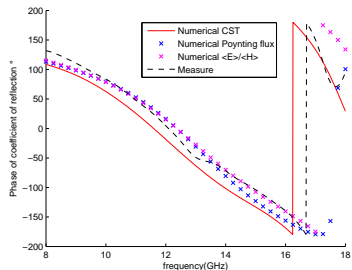
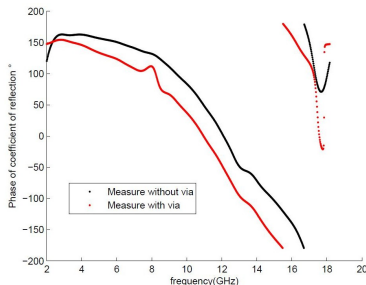
Sievenpiper

$$Z_{\text{eff}} = \frac{j\omega L}{1 - LC\omega^2}$$



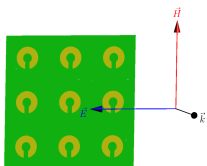
| $D$ [mm] | $g$ [mm] | Measured [GHz] | Analytical [GHz] |
|----------|----------|----------------|------------------|
| 10       | 4        | 9,84           | 9,4              |
| 7        | 2        | 10             | 9,3              |
| 6        | 2        | 10,85          | 10,86            |
| 10       | 5        | 11,75          | 11,2             |

# Comparison with the simulations

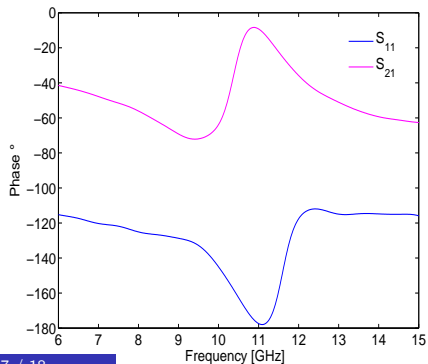
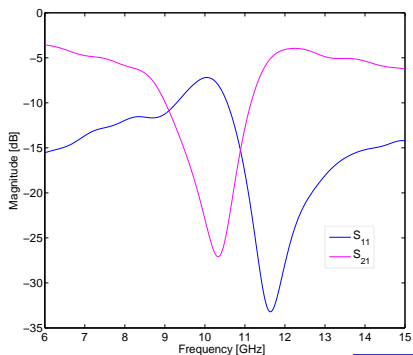


| Method                                  | Frequency [GHz] | Sample (D,g) |             |           |             |
|---|-----------------|--------------|-------------|-----------|-------------|
|   |                 | (10,4)       | (10,5)      | (7,2)     | (6,2)       |
| Measurements                            | Resonance       | 10,35        | 11,74       | 10        | 10,84       |
|   | Bandwidth       | 9,3-11,37    | 10,33-12,77 | 8-11,25   | 7,06-12,34  |
| $\langle E \rangle / \langle H \rangle$ | Resonance       | 9,9          | 11,74       | 10,3      | 11,9        |
|   | Bandwidth       | 8,8-10,8     | 10,3-12,7   | 8,9-11,7  | 10,25-13,75 |
| Poynting flux                           | Resonance       | 9,4          | 11,5        | 10,8      | 12          |
|   | Bandwidth       | 8,3-10,3     | 10,2-12,4   | 9,3-12,2  | 10,6-13,75  |
| CST                                     | Resonance       | 9,54         | 11,23       | 10,1      | 11,82       |
|   | Bandwidth       | 8,56-10,42   | 10-12,17    | 8,7-11,42 | 9,97-13,76  |

# SRR characterization : $r_{int} = 1 \text{ mm}$ , $r_{ext} = 2,5 \text{ mm}$

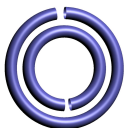


| d (mm) | Resonance frequency [GHz] |            |                |
|--------|---------------------------|------------|----------------|
|        | Measured                  | Analytical | Homogenization |
| 0,8    | 9,4                       | 9,37       | 9,47           |
| 0,9    | 9,6                       | 9,94       | 10,05          |
| 1      | 10,32                     | 10,47      | 10,59          |



# Conclusions and outlook

- Homogenization of SRR and HIS.
- Realization of a compact characterization system.
- The S-parameters are filtered in the time domain.



- Computation  $\epsilon_{eff}$  (split-ring).
- Homogenization of SRR based structures.
- Homogenization of metallic wires arrays.
- Gated Reflect Line calibration.